

Bridging **High-Level Synthesis** and **Application-Specific Arithmetic**: The Case Study of Floating-Point Summations

Yohann Uguen **Florent de Dinechin** Steven Derrien

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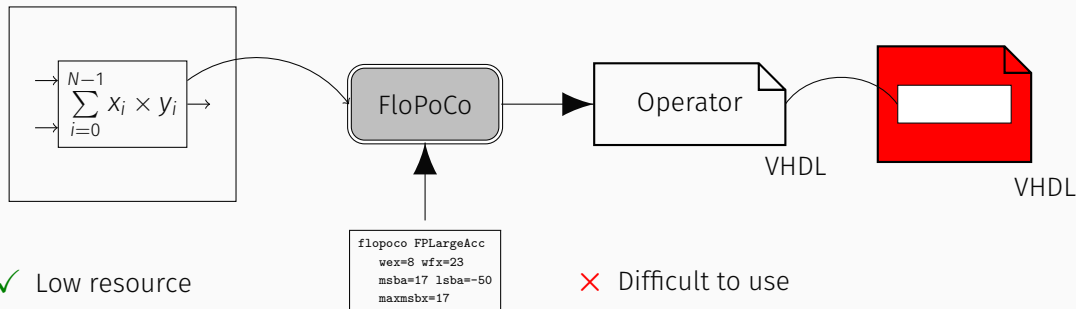
University of Lyon, INSA Lyon, Inria, CITI
F-69621 Villeurbanne, France
{Yohann.Uguen, Florent.de-Dinechin}@insa-lyon.fr

University Rennes 1, IRISA
Rennes, France
Steven.Derrien@univ-rennes1.fr

Circuit: from specification to implementation?



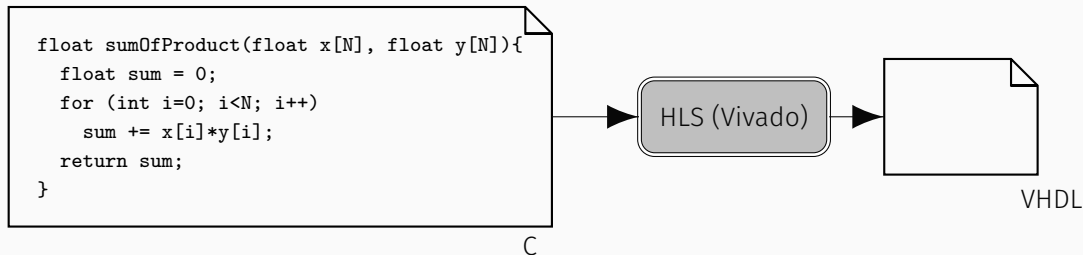
Arithmeticians point of view: highly tuned hardware, custom datatypes



- ✓ Low resource
- ✓ Low latency
- ✓ Accuracy control

- ✗ Difficult to use
 - operator nature
 - FloPoCo's interface
 - integration of the operator

Compiler designers point of view: follow language semantics and datatypes



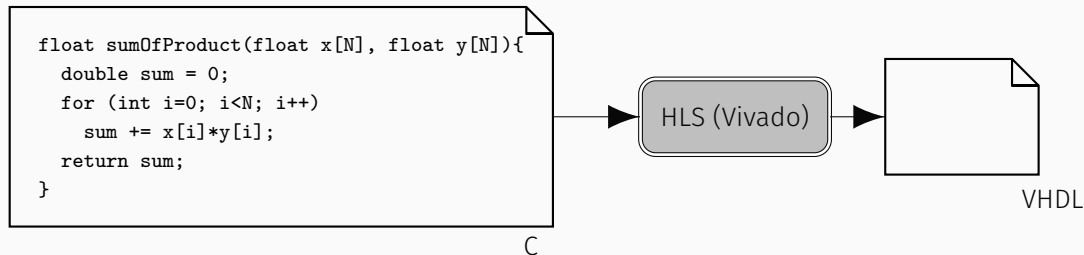
✓ Ease of use

✗ Moderate resource

✗ Moderate latency

✗ No accuracy control

Compiler designers point of view: follow language semantics and datatypes



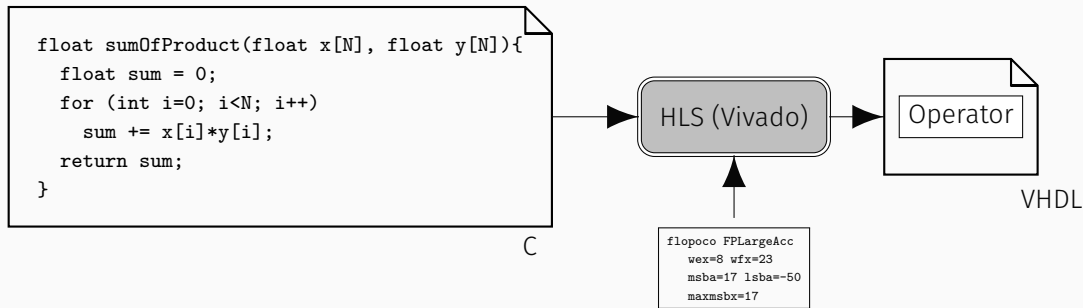
✓ Ease of use

✗ Moderate resource

✗ Moderate latency

✗ No accuracy control

Naive solution : interface FloPoCo operators with HLS



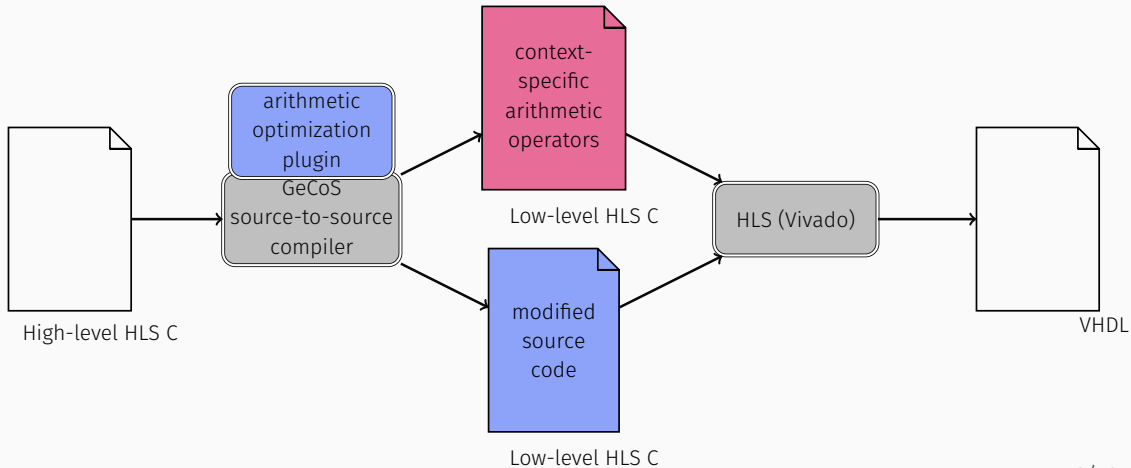
- ✓ Custom datatypes
- ✓ Better resource/accuracy control

- ✗ Vivado HLS strongly discourages it
- ✗ Sub-optimal compiler optimizations
- ✗ Can't simulate circuit in C

Proposed method : embed FloPoCo's spirit in HLS compilers

- ✓ Ease of use
 - ✓ Low resource
 - ✓ Low latency
 - ✓ Accuracy control
-
- Transform variable datatypes when specified
 - Better than custom datatypes: a high-level pragma
 - Vivado HLS compliant

HLS tools can perform IR optimizations



The arithmetic side:

Exact accumulator in Vivado HLS

The compiler side:

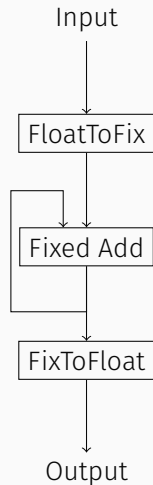
Source-to-source transformations using GeCoS

Evaluation

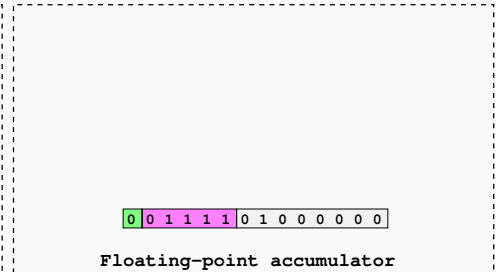
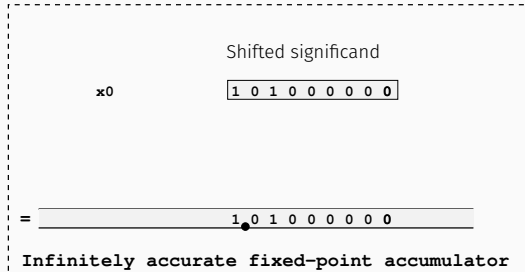
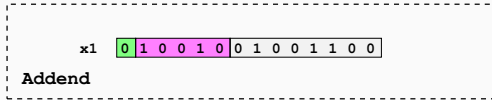
Conclusion

THE ARITHMETIC SIDE: EXACT ACCUMULATOR IN VIVADO HLS

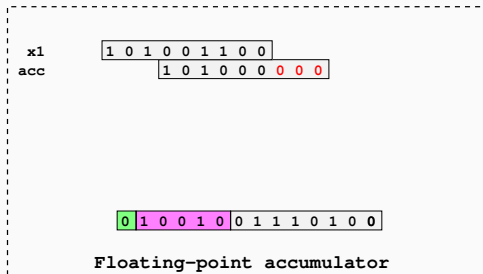
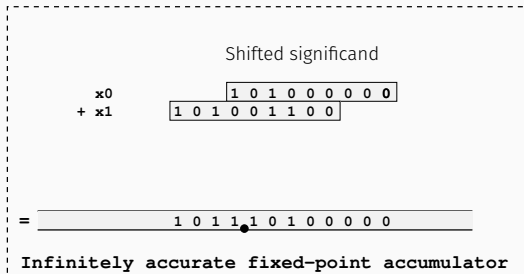
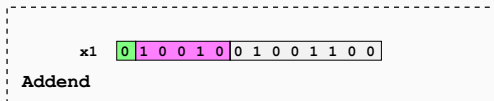
- Fixed-point accumulator:
custom application specific format
- Based on [An FPGA-specific approach to floating-point accumulation and sum-of-products, 2008]
- Minor improvements
 - support for sub-normals
 - increased modularity
- Each box tailored to application requirements



FIXED-POINT VS FLOATING POINT ACCUMULATOR



FIXED-POINT VS FLOATING POINT ACCUMULATOR



FIXED-POINT VS FLOATING POINT ACCUMULATOR

x2 0 1 0 1 0 0 0 0 0 1 1 0 0 1
Addend

x0
+ x1

Shifted significand

1 0 1 0 0 0 0 0 0

1 0 1 0 0 1 1 0 0

= 1 0 1 1 . 1 0 1 0 0 0 0 0

Infinitely accurate fixed-point accumulator

0 1 0 0 1 0 0 1 1 1 0 1 0 0

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR

x2 0 1 0 1 0 0 0 0 0 1 1 0 0 1
Addend

Shifted significand

x0 1 0 1 0 0 0 0 0 0
+ x1 1 0 1 0 0 1 1 0 0
+ x2 1 0 0 0 1 1 0 0 1

= 1 0 1 1 1 0 . 1 1 0 0 0 0 0 0

Infinitely accurate fixed-point accumulator

x2 1 0 0 0 1 1 0 0 1
acc 1 0 1 1 1 0 1 0 0

0 1 0 1 0 0 0 1 1 1 0 1 1 0

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR

x3	0	0	1	1	0	1	0	0	1	0	1	1	1	1
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Addend

Shifted significand

$$\begin{array}{r} \mathbf{x0} \quad \quad \quad 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ +\ \mathbf{x1} \quad \quad 1\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 0 \\ +\ \mathbf{x2} \quad 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1 \end{array}$$

$$= \underline{\underline{1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0}}$$

Infinitely accurate fixed-point accumulator

0	1	0	1	0	0	0	1	1	1	0	1	1	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR

x3 0 0 1 1 0 1 0 0 1 0 1 1 1 1
Addend

Shifted significand

x0	1 0 1 0 0 0 0 0 0
+ x1	1 0 1 0 0 1 1 0 0
+ x2	1 0 0 0 1 1 0 0 1
+ x3	1 0 0 1 0 1 1 1 1

= 1 0 1 1 1 0 . 1 1 1 0 0 1 0 1 1 1 1

Infinitely accurate fixed-point accumulator

x3
acc 1 0 1 1 1 0 1 1 0 1 0 0 1 0 1 1 1 1

0 1 0 1 0 0 0 1 1 1 0 1 1 1

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR

x4 0 1 0 0 0 0 1 0 1 0 0 1 0 0
Addend

Shifted significand

x0 1 0 1 0 0 0 0 0 0
+ x1 1 0 1 0 0 1 1 0 0
+ x2 1 0 0 0 1 1 0 0 1
+ x3 1 0 0 1 0 1 1 1 1

= 1 0 1 1 1 0 . 1 1 1 0 0 1 0 1 1 1 1

Infinitely accurate fixed-point accumulator

0 1 0 1 0 0 0 1 1 1 0 1 1 1

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR

x4 0 1 0 0 0 0 1 0 1 0 0 1 0 0
Addend

Shifted significand

x0 1 0 1 0 0 0 0 0 0
+ x1 1 0 1 0 0 1 1 0 0
+ x2 1 0 0 0 1 1 0 0 1
+ x3 1 0 0 1 0 1 1 1 1
+ x4 1 1 0 1 0 0 1 0 0

= 1 1 0 0 1 0 . 0 0 1 0 1 1 0 1 1 1 1

Infinitely accurate fixed-point accumulator

x4 acc 1 1 0 1 0 0 1 0 0
1 0 1 1 1 0 1 1 1

0 1 0 1 0 0 1 0 0 1 0 0 0 1

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR

x5 0 0 1 0 1 0 0 1 1 1 0 0 1 0
Addend

Shifted significand

x0 1 0 1 0 0 0 0 0 0
+ x1 1 0 1 0 0 1 1 0 0
+ x2 1 0 0 0 1 1 0 0 1
+ x3 1 0 0 1 0 1 1 1 1
+ x4 1 1 0 1 0 0 1 0 0

= 1 1 0 0 1 0 0 0 1 0 1 1 0 1 1 1 1

Finite accuracy fixed-point accumulator

x4 acc 1 1 0 1 0 0 1 0 0
1 0 1 1 1 0 1 1 1

0 1 0 1 0 0 1 0 0 1 0 0 0 1

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR

x5 0 0 1 0 1 0 0 1 1 1 0 0 1 0
Addend

Shifted significand

x0	1 0 1 0 0 0 0 0 0
+ x1	1 0 1 0 0 1 1 0 0
+ x2	1 0 0 0 1 1 0 0 1
+ x3	1 0 0 1 0 1 1 1 1
+ x4	1 1 0 1 0 0 1 0 0
+ x5	1 0 1 1 1 0 0 1 0

= 1 1 0 0 1 0 0 0 1 1 0 0 1 1 1 0 1 0

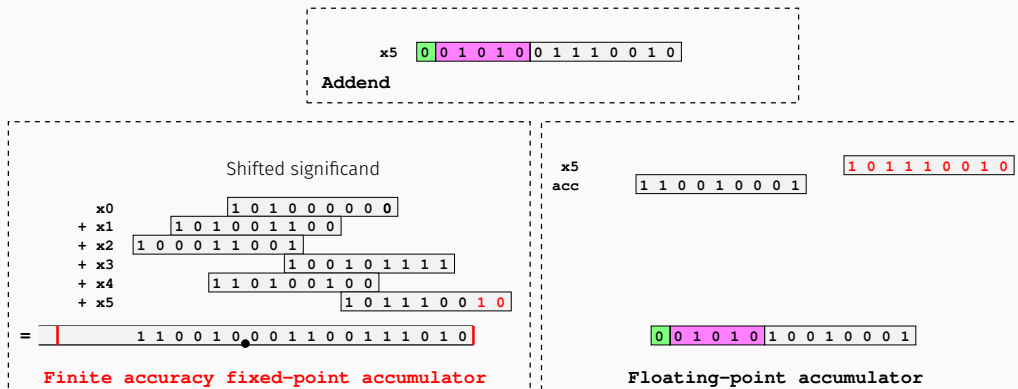
Finite accuracy fixed-point accumulator

x5
acc 1 1 0 0 1 0 0 0 1 1 0 1 1 1 0 0 1 0

0 0 1 0 1 0 1 0 0 1 0 0 0 1

Floating-point accumulator

FIXED-POINT VS FLOATING POINT ACCUMULATOR



Accuracy:

Exact Result = 50.2017822265625

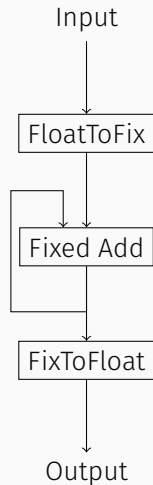
FP Acc = 50.125

Fixed-Point Acc = 50.20166015625

Vivado HLS compliant C

Same results as FloPoCo operator in ~ 50 lines of code in terms of:

- LUTs
- Registers
- DSPs



THE COMPILER SIDE: SOURCE-TO-SOURCE TRANSFORMATIONS USING GECOS

Use of a pragma

```
#define N 100000
float computeSum(float in1[N], float in2[N]){
    float sum = 0;
    for (int i=1; i<N-1; i++){
        sum += in1[i]*in2[i-1];
        sum += in1[i];
        sum += in2[i+1];
    }
    return sum;
}
```

C

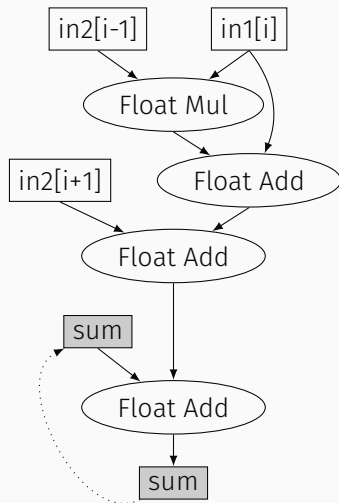
Use of a pragma

```
#define N 100000
float computeSum(float in1[N], float in2[N]){
    float sum = 0;
    #pragma FPacc VAR=sum MaxAcc=300000.0 epsilon=1e-15
    for (int i=1; i<N-1; i++){
        sum += in1[i]*in2[i-1];
        sum += in1[i];
        sum += in2[i+1];
    }
    return sum;
}
```

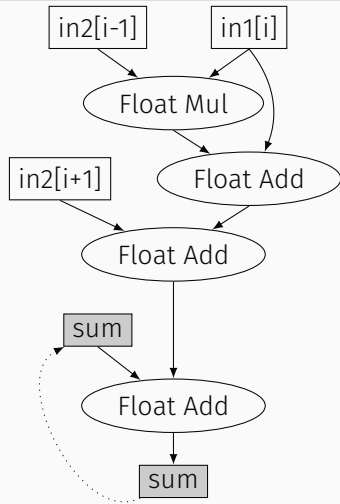
Build a **GeCoS** plug-in

- Read C code
- Transform to compiler IR
- Retrieve DFG associated to pragma
- Apply transformations
- Regenerate C code

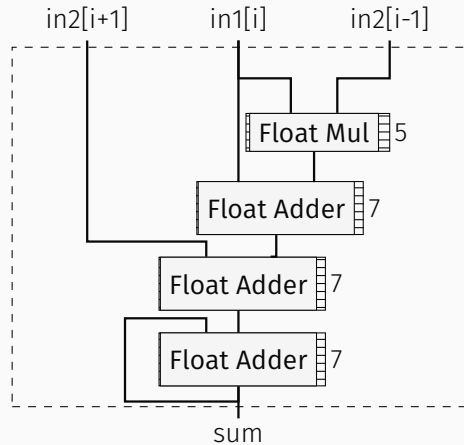
```
sum += in1[i]*in2[i-1];  
sum += in1[i];  
sum += in2[i+1];
```



ORIGINAL DFG - GENERATED CIRCUIT

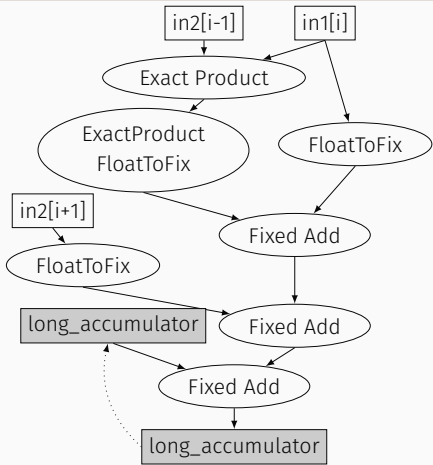


DFG

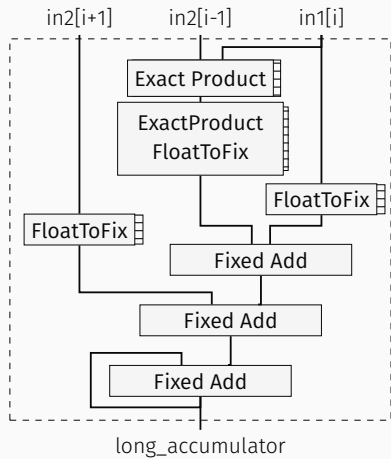


Circuit

TRANSFORMED DFG - GENERATED CIRCUIT



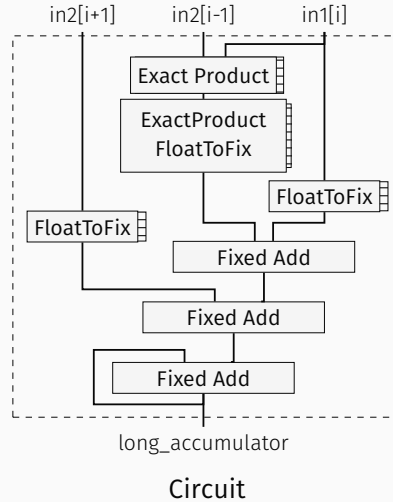
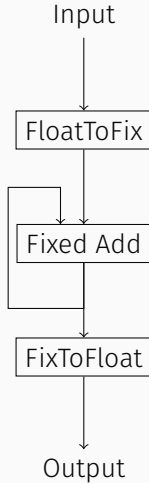
DFG



Circuit

This **CANNOT** be obtained using FloPoCo

TRANSFORMED DFG - GENERATED CIRCUIT



This **CANNOT** be obtained using FloPoCo

EVALUATION

Evaluation of the previous example

Single cycle accumulation improves latency

- After place-and-route
- For 100MHz
- On Xilinx Kintex 7
- Using Vivado HLS 2016.3
- Using Vivado 2016.3

	Original no pragma	Transformed epsilon=1e-7	Transformed epsilon=1e-10	Transformed epsilon=1e-19
LUTs	538	693	824	1400
DSPs	5	2	2	2
Latency (N=100K)	2000K	100K	100K	100K

FP benchmark suite

- 10 kernels
- 5 contains accumulations

Methodology

- Implemented a golden reference (MPFR) for accuracy
- Synthesized the 5 kernels w/ and w/o transformations
- Accumulator tailored to be just more accurate than original benchmarks

Three types of results

1. One kernel gets +33% LUT usage, -60% DSPs for a 400% speed improvement
2. Two kernels different loop sizes: between +24% and +26% LUT usage, -60% DSPs for a speed improvement between 480% and 840%
3. Two kernels have less than 1% difference for resource usage but unpredictable latency
 - Better latency observed by simulation
 - Benchmarks should be completely re-written to be suitable for HLS

CONCLUSION

Arithmetic optimizations and HLS are compatible

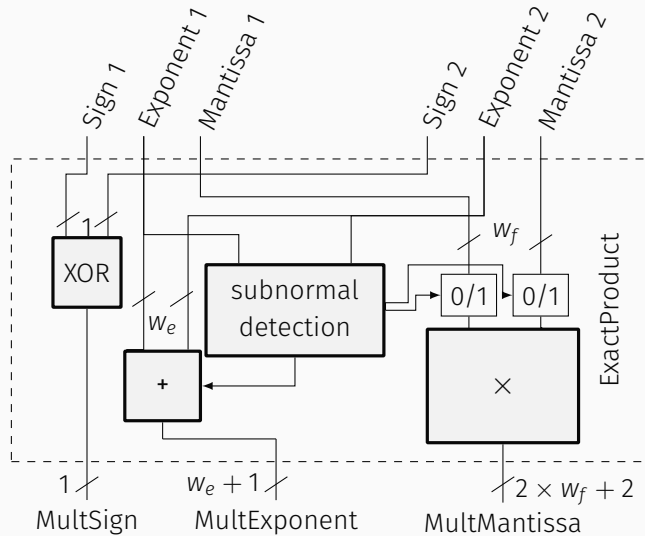
- At least for sums-of-products
- Tool distributed with GeCoS
 - `http://gecos.gforge.inria.fr/doku/doku.php?id=subprojects:arithmetic\_optimizations\_for\_hls`

Future work

- Support more operators
- Make more use of compiler knowledge such as loop counts when possible
- Integration to a HLS tool

Thank you

EXACT MULTIPLIER



FPMARK RESULTS

Benchmark	Type	LUTs	DSPs	Latency	Accuracy
Livermore	Original	384	5	80K	11 bits
	Transformed	576 (+33%)	2 (-60%)	20K (/4)	13 bits
LU-8	Original	809	5	82	8-23 bits
	Transformed	1007 (+24%)	2 (-60%)	17 (/4.8)	23 bits
LU-45	Original	819	5	452	8-23 bits
	Transformed	1034 (+26%)	2 (-60%)	54 (/8.4)	23 bits
Scholes	Original	15640	175	N/A	19 bits
	Transformed	15923 (+1%)	175 (+0%)	N/A	23 bits
Fourier	Original	34596	64	N/A	6 bits
	Transformed	34681 (+1%)	59 (-8%)	N/A	11 bits

ACCUMULATOR - ARCHITECTURE

